ENCYCLOPEDIA on Early Childhood Development

NUMERACY

# Numerical Knowledge in Early Childhood

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## Introduction

Research on the numerical knowledge of young children has grown rapidly in recent years. This research encompasses a wide range of abilities and concepts, from infants' ability to discriminate between collections containing different numbers of elements[1,2] to preschoolers' understanding of number words[3,4] and counting,[5,6,7] and their grasp of the inverse relation between addition and subtraction.[8,9]

## Subject

Research on young children's numerical knowledge provides an important foundation for the formulation of standards for early childhood education[10] and for the design of early childhood mathematics curricula.[11,12,13] Further, the mathematics knowledge that children acquire before they begin formal schooling has important ramifications for school performance and future career options.[14] An analysis of predictors of academic achievement, based on six longitudinal data sets, found that children's math skills at school entry predicted subsequent school performance more strongly than did early reading skills, attentional skills or socioemotional skills.[15]

## Problems

Fundamentally, numeracy entails understanding numbers as representations of a particular kind of magnitude. Correspondingly, understanding the development of numeracy in early childhood entails understanding both how children come to understand the basic quantitative relations that numbers share with other kinds of quantities and how they come to understand the aspects of number that distinguish it from other kinds of quantities.

## Research Context

Piaget's classic research on logico-mathematical development investigated children's understanding of general properties of quantity such as seriation and the conservation of equivalence relations under certain kinds of transformations.[16] His view, however, was that this kind of knowledge emerges only with the acquisition of concrete-operational thinking, around 5-7 years of age. Subsequent researchers[17] undertook to demonstrate that younger children have considerably more numerical knowledge than Piaget recognized; and contemporary research provides evidence of a wide range of early numerical abilities.[18]

## Key Research Questions

An influential but controversial claim in current research literature on early numerical abilities holds that the brain is "hard wired" for number.[19,20] This idea is often supported by evidence of numerical discrimination by human infants and by animals.[21] Critics of innatist (philosophical doctrine that holds that the mind is born with ideas/knowledge) accounts of numerical knowledge, however, note the pervasiveness of developmental change in numerical reasoning,[22] the slow differentiation of number from other quantitative dimensions,[23] and the contextualized nature of early numerical knowledge.[24] Further, accumulating evidence indicates that language[24] and other cultural products and practices[25,26] make enormous contributions to young children's acquisition of numerical knowledge.

## Recent Research Results

### *Numerical knowledge in infancy*

One of the most active areas of current research concerns the numerical abilities of infants. Kobayashi, Hiraki and Hasegawa[1] used discrepancies between visual and auditory information about the number of items in a collection to test for numerical discrimination in six-month-olds.

They showed infants objects that made a sound when dropped onto a surface, and then dropped two or three of the objects behind a screen so that the infants heard the tone each item made but could not see the items. They then removed the screen to reveal either the correct number of objects or a different number (3 if there had been 2 tones, and vice versa). Infants looked longer when the number of items revealed did not match the number of tones, indicating that they were able to distinguish between two and three items. Other research indicates that six-month-old infants can also discriminate between larger numerical quantities, provided the numerical ratio between them is large. Six-month-old infants discriminate between 4 vs.8[27] and even 16 vs. 32.[28] When the contrast is reduced (for example, 8 vs. 12), however, six-month-old infants fail[29] but older ones succeed.[2] Thus, infants become able to make finer numerical discriminations as they get older.

*Young children's knowledge about numerical relations*

Because numbers represent a kind of magnitude, a fundamental aspect of numerical knowledge pertains to equal, less-than and greater-than relations between numerical quantities.[30] Somewhat surprisingly, in light of the infancy findings, it is a significant developmental achievement for preschool children to compare sets numerically, particularly when that entails disregarding other differences between the sets.

For example, Mix[31] studied the ability of three-year-olds to numerically match a set of 2, 3 or 4 black dots. This task was easy when the manipulatives children were given were perceptually similar to the dots they were to match (e.g., black disks, or red shells about the same size as the dots). However, children's performance dropped when the manipulatives contrasted perceptually with the dots (e.g., lion figurines or heterogeneous objects).

Muldoon, Lewis, and Francis[7] assessed four-year-olds' ability to evaluate the numerical relation between two rows of blocks (with 6-9 items per row) in the face of misleading length cues, that is, when two unequal-length rows contained the same number of items, or two equal-length rows contained different numbers of items.  Most children relied on length comparisons rather than on counting the items to compare the rows. However, a three-session training procedure led to better performance, particularly among children who, as part of the training, were asked to explain why the rows were in fact numerically equal or unequal (as indicated by the experimenter).

**Research Gaps**

While experimental data concerning early numeracy is accumulating rapidly, the absence of theoretical accounts that incorporate the full range of empirical results limits our understanding of how the diverse findings already obtained fit together and what issues remain to be resolved. In the infancy literature, for example, competing accounts of early numerical abilities have generated much research in the past few years, yet the findings have not lessened the theoretical controversy. In advancing theoretical conclusions, researchers need to be cognizant of the entire corpus of findings, and their theories need to be formulated precisely enough that they can be differentiated empirically.

In addition, researchers need to gather better information about the processes that lead to advances in early numeracy knowledge. We know that young children's performance is affected by contextual variables ranging from culture and social class[32] to patterns of parent-child[33,34] and teacher-child[35] interaction. As yet, however, we have only small pieces of information, mostly from experimental training studies[7,25,36] about how particular experiences alter children's numerical thinking. Research that provides converging data about (a) young children's everyday numerical experiences, and how they vary with the age of the child, and (b) the experimental effects of those kinds of experiences on children's thinking, would be especially helpful.

## Conclusions

The available research on young children's developing knowledge about number supports four generalizations that have important implications for policy and practice. First, numerical development is multifaceted. Early childhood numeracy encompasses much more than counting and knowing some elementary arithmetic facts. Second, notwithstanding the number-related abilities evidenced even by infants, age-related change is pervasive. In age group comparisons, the older children nearly always perform better. Third, variability is pervasive. Individual children vary in their performance across different numerical tasks,[37] in their engagement in particular sorts of numerical reasoning across different contexts,[3] and even in their trial-to-trial responses within a single task.[5,38] Finally, children's progress in acquiring numerical knowledge is highly malleable. It is influenced by informal activities such as playing board games,[25] by experimental activities designed to illuminate numerical relationships,[7,36] and by variations in the ways in which parents[33,34] and teachers[35] talk to children about numbers.

## Implications

An important contribution that research on early childhood numeracy can make to policy and practice is to inform the goals we set for early mathematics instruction. Just as numerical development in early childhood is multi-faceted, the goals of early childhood instructional programs should be much broader than enhancing children's counting skills or teaching them some basic arithmetic facts. Numbers, like other kinds of magnitudes, are characterized by relations of equality and inequality. At the same time, they differ from other kinds of magnitudes in that they are based on the partitioning of an overall quantity into units. Instructional activities that encourage children to think about relationships between quantities and effects of transformations such as partitioning, grouping, or rearranging those relationships may be helpful in advancing children's understanding of these ideas. The variability and malleability of young children's numerical thinking indicate the potential for early childhood instructional programs to contribute substantially to children's growing knowledge about numbers.

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